The document contains details for power analysis for a simple linear regression model under a longitudinal data setting when the hypothesis is expressed in terms of an effect size (slope coefficient divided by standard deviation).

Linear Regression with Growth Curve Modeling

As in Tu et al. (2004), the class of linear regression models for repeated measures is given by:

$$E\left(y_{it} \mid \mathbf{x}_{it}\right) = \beta_0 + t\beta_1, \quad Var\left(y_{it} \mid \mathbf{x}_{it}\right) = \sigma^2, \quad 0 \leq t \leq m, 1 \leq i \leq n. \tag{1}$$

This class of models is quite popular in growth curve analysis. The GEE estimate $\widehat{\boldsymbol{\beta}}$ can be expressed in closed form:

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{i=1}^{n} X_i^\top V^{-1} X_i\right)^{-1} \left(\sum_{i=1}^{n} X_i^\top V^{-1} \mathbf{y}_i\right), \tag{2}$$

where $X_i = \left(\mathbf{x}_{i1}, \ldots, \mathbf{x}_{im}\right)^\top$. Then, we have:

$$\sqrt{n}\left(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right) \rightarrow_d N\left(0, \Sigma_{\boldsymbol{\beta}} = E^{-1}\left(X_1^\top V^{-1} X_1\right)\right), \tag{3}$$

Now, let $\mathbf{x}_{it} = (1, t)^\top$ $(0 \leq t \leq m)$ and $V = \sigma^2 C(\rho)$. Then, $X_i = X$ and

$$\begin{aligned} \Sigma_{\boldsymbol{\beta}} &= E^{-1}\left(X_1^\top V^{-1} X_1\right) = E^{-1}\left[X^\top \sigma^{-2} C^{-1}(\rho) X\right] \\ &= \sigma^2 \left[X^\top C^{-1}(\rho) X\right]^{-1} = \sigma^2 G(m, \rho), \end{aligned} \tag{4}$$

where $G(m, \rho) = \left[X^\top C^{-1}(\rho) X\right]^{-1}$. We then obtain from (3) and (4) that

$$\sqrt{n}\left(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right) \rightarrow_d N\left(0, \sigma^2 G(m, \rho)\right). \tag{5}$$

Consider testing the hypothesis:

$$H_0 : \beta_1 = 0 \quad \text{vs.} \quad H_0 : \beta_1 = b \tag{6}$$

Let $K = (0, 1)$. Then, the above can be expressed as a linear contrast as follows:

$$H_0 : K\boldsymbol{\beta} = 0, \quad H_a : K\boldsymbol{\beta} = b. \tag{7}$$

It follows from standard asymptotic theory and (5) that $K\widehat{\boldsymbol{\beta}}$ has a limiting normal distribution under either H_0 or H_a, i.e.,

$$H_0 : \sqrt{n} K\widehat{\boldsymbol{\beta}} \rightarrow_d N\left(0, K\Sigma_\beta K^\top\right), \quad H_a : \sqrt{n}\left(K\widehat{\boldsymbol{\beta}} - \mathbf{d}\right) \rightarrow_d N\left(0, K\Sigma_\beta K^\top\right). \tag{8}$$

In addition, the centered quadratic statistic, $Q_{n0}^2 = n\left[K\left(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right)\right]^\top \left(K\Sigma_\beta K^\top\right)^{-1}\left[K\left(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right)\right]$, has an asymptotic central χ^2 distribution, $Q_{n0}^2 \to_d \chi_s^2(0)$, where $\chi_s^2(c)$ denotes a χ^2 distribution with degree of freedom s and non-centrality parameter c. In other words, under H_0 and H_a, the quadratic or Wald statistic, $Q_n^2 = n\left(K\widehat{\boldsymbol{\beta}}\right)^\top \left(K\Sigma_\beta K^\top\right)^{-1}\left(K\widehat{\boldsymbol{\beta}}\right)$, has approximately a central and non-central χ_s^2 distribution, respectively,

$$H_0 : Q_n^2 \sim \chi_s^2(0), \qquad H_a : Q_n^2 \sim \chi_s^2(c), \tag{9}$$

where $c = nb^2\left(K\Sigma_\beta K^\top\right)^{-1} = n\left(\frac{b}{\sigma}\right)^2\left(KG(m,\rho)K^\top\right)^{-1}$. The quantity, $\frac{b}{\sigma}$, will be denoted as the effect size. In particular, if $\sigma = 1$, b in (6) is the effect size.