Complex, Pulsatile Mathematical Model of Arteriovenous Fistula Hemodynamics to Predict Factors Contributing to Steal After Initial Fistula Placement

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INTRODUCTION:

Steal after initial arteriovenous fistula (AVF) placement results in significant morbidity and access failure. Robust mathematical modeling can be used to study and predict changes in hemodynamics. We propose a pulsatile mathematical model of the upper extremity to predict steal based on arterial, fistula, and venous geometries.

BACKGROUND:

Electrical analogs of the upper extremity have been described using both the Wheatstone Bridge model, which accounts for collateral flow, and series type circuit models.[1,2,3] Equations derived from the Wheatstone bridge (*Figure #1*) predict the magnitude of **mean non-pulsatile flow** based on vessel resistance.

Anterograde Flow in Distal Segment:

$$\frac{R_{fistula}}{R_{proximal}} > \frac{\left(\frac{R_{Radial} * R_{Ulnar}}{R_{Radial} + R_{Ulnar}}\right)}{R_{Collateral}}$$

No Flow in Distal Segment:

$$\frac{R_{fistula}}{R_{proximal}} = \frac{\left(\frac{R_{Radial} * R_{Ulnar}}{R_{Radial} + R_{Ulnar}}\right)}{R_{Collateral}}$$

Retrograde Flow in Distal Segment:

$$\frac{R_{fistula}}{R_{proximal}} < \frac{\left(\frac{R_{Radial} * R_{Ulnar}}{R_{Radial} + R_{Ulnar}}\right)}{R_{Collateral}}$$

Figure #1: Derivation of mean flow in the distal segment based on DC-circuit analysis. [1,2]

Reports have shown that retrograde **mean flow** in the distal segment is neither necessary nor sufficient to predict steal. [2] We hypothesize that a pulsatile time-based model, which allows for iterative simulation of hemodynamics based on variable vessel geometry, will show **instantaneous flows** in the distal segment conducive to future distal ischemia.

METHODS:

- A circuit-based Simulink® **three-element arterial Windkessel** model of the upper extremity was created, incorporating a pulsatile arterial input with arterial **capacitance** and **impedance** (resistance). Typical anatomic vascular dimensions (*Table#1*) were converted using basic fluid modeling equations (*Figure #2*). [4,5,6,7,8]

Artery Name	Artery Radius (cm)	Artery Length (cm)	Calculated Resistance mmHg/(cm³/min)	Calculated Elastic Capacitance (cm³/mmHg)
Axillary (Inflow)	0.25	10	0.0943	0.00290
Proximal	0.20	10	0.4775	0.00290
Distal	0.20	0.30	0.1430	0.00004
Radial	0.15	15	2.2635	0.00090
Ulnar	0.15	15	2.2635	0.00090
Fistula	0.30	10	0.1886	0.00090
Venous*			3	1

*Venous Resistance and Capacitance Chosen based on Experimental and Empirical Model Data

Table #1: Experimental Parameters of Upper Arm Model .[7]

$$Laminar\ Vascular\ Resistance = \frac{128 * \mu * L}{\pi * (2 * r)^4}$$

$$Vascular\ Capacitance = \frac{2 * r * (\pi * r^2 * L)}{t_w * E}$$

Constants:
μ = Blood's Dynamic Viscosity = 4 (milliPascal seconds @37°C)
t_w = Arterial Vessel Wall Thickness = 0.1 (centimeters)
E = Arterial Wall Elastic Modulus = 455 (kilopascal)

Variables:
r = Vessel Internal Radius (centimeters)
L = Vessel Length (centimeters)

Figure #2: Basic dynamic fluid modeling equations. [5,6]

- Collateral flow was calculated as 10% of arterial inflow.
- The pulsatile input was modeled using a 1 Hz sine-wave current generator. In order to condition the signal from the sine-wave generator to output physiologic flow, two diodes were added to the model (*Figure #3*).

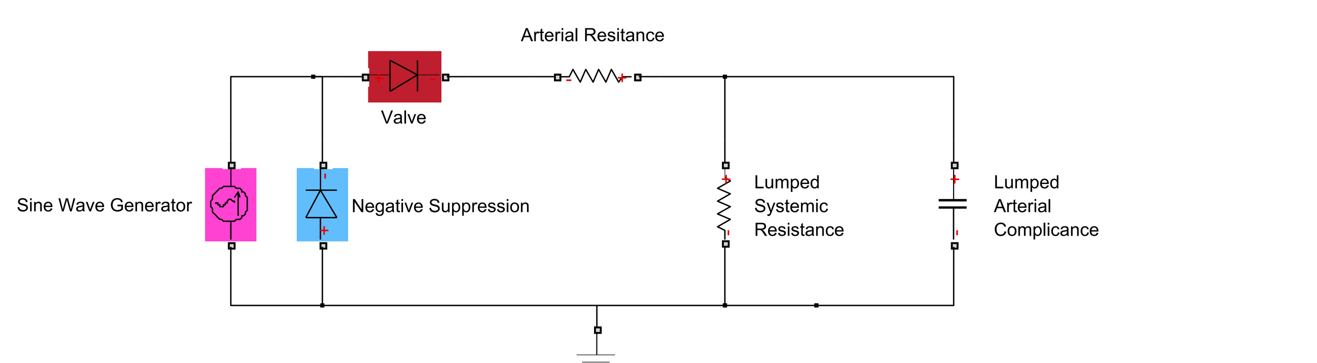

Figure #3: Lumped Three-element Windkessel model and sine-wave current driver with signal conditioning diodes. [4] *The negative suppression diode removes the negative component of the sine-wave and the valve diode prevents back flow during diastole.*

METHODS (continued) :

- The model in *Figure #3* produced both physiologic arterial pressure waveforms (*Figure #4A*) and an instantaneous flow waveform with resulting mean arterial flow of 1002.6 cc/min (*Figure #4B*).

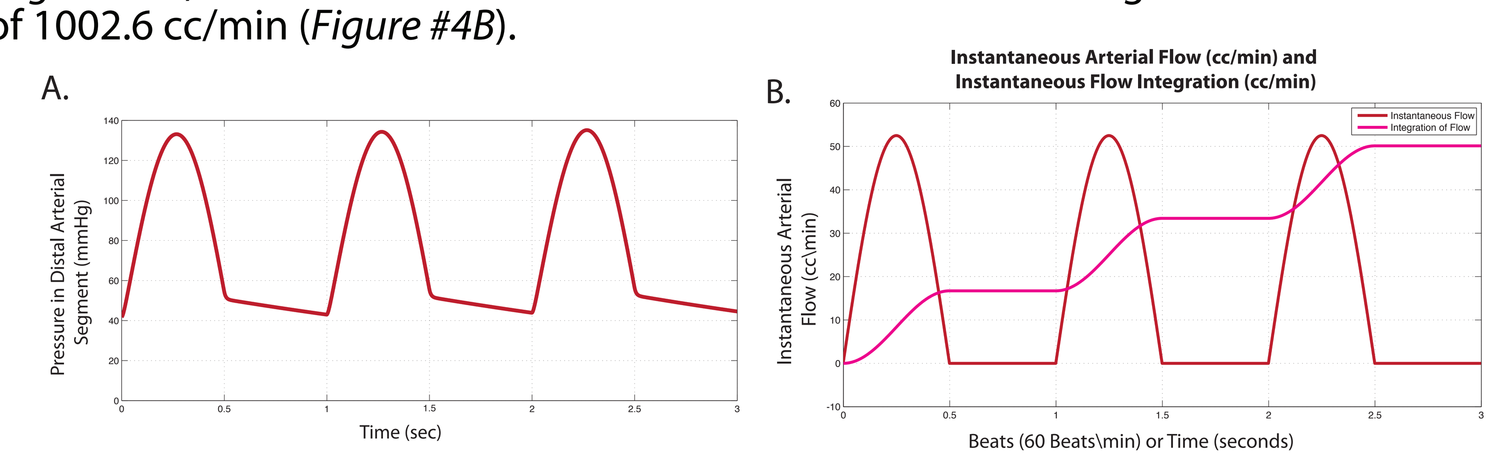

Figure #4: A . Physiologic arterial pressure waveform;
B. Instantaneous arterial flow waveform and integration of flow waveform.

- With the completed Upper Arm AVF Simulink® model (*Figure #5C*) two experiments were conducted.

Experiment #1: Effects of Fistula Radius on Distal Arterial Hemodynamics: Observed hemodynamic properties of the distal arterial segment and the AVF as the radius of the AVF was iteratively changed from 0.05 cm to 0.5 cm.

Experiment #2: Effects of Radial/Ulnar Radius on Distal Arterial Hemodynamics: Observed hemodynamic properties of the distal arterial segment as the radius of both the radial and ulnar artery where simultaneously varied between 0.025 cm and 0.2 cm.

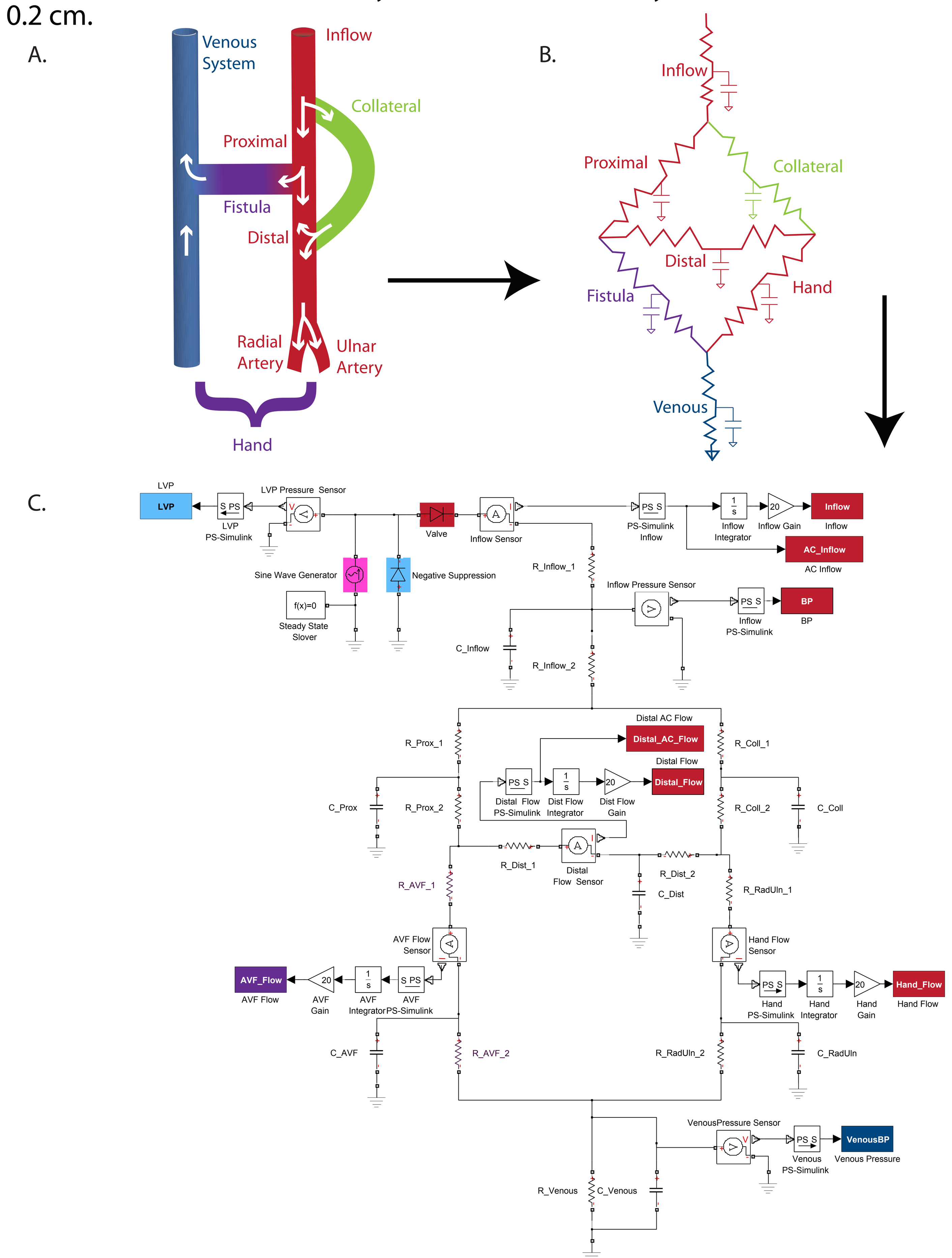

Figure #5. A. Stylized diagram of Upper Arm AVF ;
B. Three-element Windkessel Circuit diagram of Upper Arm AVF;
C. Simulink® model used to simulate pressure and flows within the AVF circuit.

RESULTS:

Experiment #1: Effects of Fistula Radius on Distal Arterial Hemodynamics:

- With a pulsatile mean arterial inflow pressure of 63 mmHg , *Figure #6A* shows that at an AVF radius of 0.285 cm (diameter of 5.7 mm) or AVF:proximal brachial artery diameter ratio of > 1.4 resulted in reversed mean flow in the distal arterial segment. This relationship corresponds to the calculated mean reversal of flow in the distal segment as calculated from equation in *Figure#1*.
- *Figure #6C* demostrates oscillatory flow in the distal arterial segment when the AVF radius > 0.15 cm (diameter of 3 mm) but < 0.285 cm (diameter of 5.7 mm) or AVF:proximal brachial artery diameter ratio of > 0.75 but < 1.4.
- During oscillatory flow in the distal segment, *Figure #6D* shows a mean anterograde flow in the distal segment. This finding supports that flow as detected by a time averaging flow meter would be anterograde while instantaneous flow is oscillatory. This oscillatory flow in the distal segment may promote distal ischemia or distal stenosis.
- Oscillatory flow begins when the flow in the fistula exceeds the flow delivered to the distal arterial segment when collateral flow is fixed (*Figure #6D*).

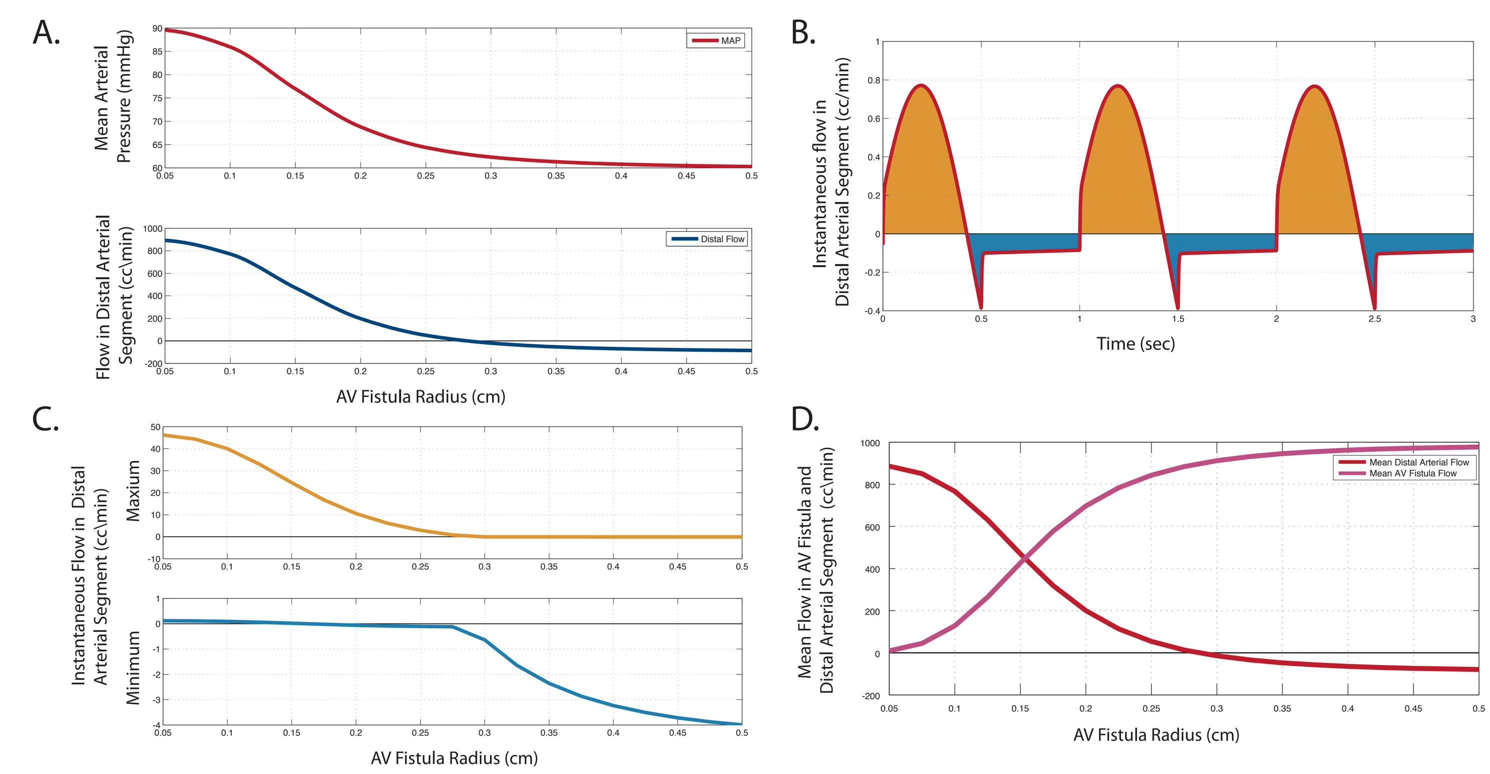

Figure #6 A. AVF radius from 0.05 cm to 0.5 cm with resulting mean arterial pressure and mean flow in distal arterial segment;
B. Instantaneous flow in the distal segment, for an AVF radius of 0.285 cm, when anterograde flow (gold area under the curve) equal retrograde flow (blue area under the curve);
C. AVF radius with resulting minimum and maximum instantaneous flow. Note flow goes from anterograde to oscillatory when blue waveform crosses zero and oscillatory to retrograde when gold line crosses zero;
D. Effect of varying AVF radius on mean flow in distal arterial segment and mean flow in AVF.

Experiment #2: Effects of Radial/Ulnar Radius on Distal Arterial Hemodynamics:

A reversed mean flow occurred in the distal arterial segment with a single vessel radius (occluded parallel artery) of ≤ 0.075 cm (diameter of 1.5 mm) or two-vessel radii ≤ 0.05 cm (diameter of 1 mm) each (Figure #7).

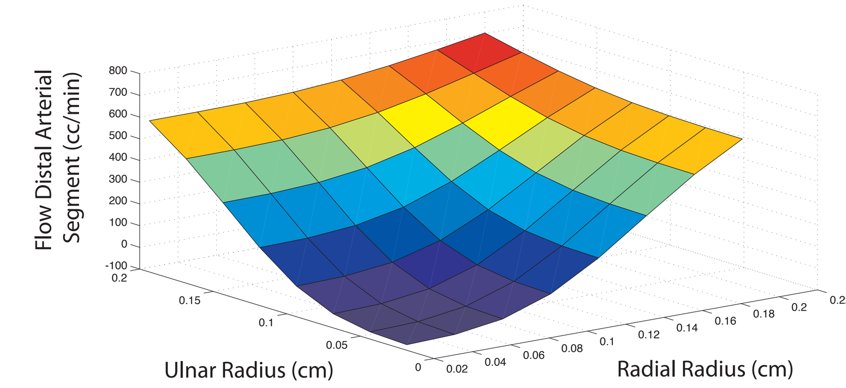

Figure #7: Effects of varying radial and ulnar radii simultaneously from 0.025 cm and 0.2 cm on mean flow in distal arterial segment.

CONCLUSIONS:

This model predicted that steal after initial AVF placement is dependent upon both the diameter of the AVF relative to the inflow artery, as well as radial and ulnar artery patency. Furthermore, it provides a continuum of data as a preoperative framework to input patient-specific physiology and dimensions to predict distal flow. Further work to clinically validate and refine this model may allow for a predictive tool to decrease or prevent the occurrence of steal after initial fistula placement.

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